MONTANA STANDARDS FOR MATHEMATICS

Mathematics is intended to give students an ability to solve problems, to communicate their ideas and strategies, and to apply their skills in other disciplines. Students are expected to understand and investigate mathematical concepts, to use mathematics in real-world situations, and to select and use appropriate technology to model and study mathematical processes.

Students will use mathematical methods to learn about six strands: Quantity (number), Algebraic Representation, Shape (geometry), Measurement, Chance and the Use of Data, and Mathematical Patterns. In every strand, it is important for all students to have a conceptual framework, a knowledge of procedures, a sense of reasonable results, and a confidence to apply their skills.

Content Standards indicate what all students should know, understand, and be able to do in a specific content area.

Benchmarks define our expectations for students' knowledge, skills, and abilities along a developmental continuum in each content area. That continuum is focused at three points—the end of grade 4, grade 8, and grade 12.

Content Standard 1 - Students engage in the mathematical processes of problem solving and reasoning, estimation, communication, connections and applications, and using appropriate technology.

Content Standard 2 - Students demonstrate understanding of and an ability to use numbers and operations.

Content Standard 3 - Students use algebraic concepts, processes, and language to model and solve a variety of real-world and mathematical problems.

Content Standard 4 - Students demonstrate understanding of shape and an ability to use geometry.

Content Standard 5 - Students demonstrate understanding of measurable attributes and an ability to use measurement processes.

Content Standard 6 - The students demonstrate understanding of an ability to use data analysis, probability, and statistics.

Content Standard 7 - Students demonstrate understanding of and an ability to use patterns, relations and functions.

Mathematics Content Standard 1

Students engage in the mathematical processes of problem solving and reasoning, estimation, communication, connections and applications, and using appropriate technology.

Rationale

These processes are essential to all mathematics and must be incorporated in all other mathematics standards.

Benchmarks

Students will:

End of Grade 4	End of Grade 8	Upon Graduation—End of Grade 12
1. solve problems from many contexts using a variety of strategies (e.g., estimate, make a table, look for a pattern, and simplify the problem). Explain the methods for solving these problems.	1. formulate and solve multi-step and nonroutine problems using a variety of strategies. Generalize methods to new problem situations.	1. recognize and formulate problems from situations within and outside mathematics and apply solution strategies to those problems.
2. apply estimation strategies throughout the problem-solving process.	2. select and apply appropriate estimation strategies throughout the problem-solving process.	2. select, apply, and evaluate appropriate estimation strategies throughout the problem-solving process.
3. communicate mathematical ideas in a variety of ways (e.g., written, verbal, concrete, pictorial, graphical, algebraic).	3. interpret and communicate mathematical ideas and logical arguments using correct mathematical terms and notations.	3. formulate definitions, make and justify inferences, express generalizations, and communicate mathematical ideas and relationships.
4. recognize and investigate the relevance and usefulness of mathematics through applications, both in and out of school.	4. recognize and investigate the relevance and usefulness of mathematics through applications, both in and out of school.	4. apply and translate among different representations of the same problem situation or of the same mathematical concept. Model connections between problem situations that arise in disciplines other than mathematics.
5. select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, and computer.	5. select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, computer, and data collection devices.	5. select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, computer, and data collection devices.

Mathematics Content Standard 2

Students demonstrate understanding of and an ability to use numbers and operations.

Rationale

An understanding of numbers and how they are used is necessary in the everyday world. Computational skills and procedures should be developed in context so the learner perceives them as tools for solving problems.

Benchmarks

Students will:

End of Grade 4	End of Grade 8	Upon Graduation—End of Grade 12
1. exhibit connections between the concrete and symbolic representation of a problem or concept.	1. use the four basic operations with whole numbers, fractions, decimals, and integers.	1. use and understand the real number system, its operations, notations, and the various subsystems.
2. use the number system by counting, grouping and applying place value concepts.	2. use mental mathematics and number sense in using order of operations, and order relations for whole numbers, fractions, decimals, and integers.	2. use definitions and basic operations of the complex number system.
3. model, explain, and use basic facts, the operations of addition and subtraction of whole numbers, and mental mathematics.	3. use the relationships and applications of ratio, proportion, percent, and scientific notation.	
4. model and explain multiplication and division of whole numbers.	4. develop and apply number theory concepts (e.g., primes, factors and multiples) in real-world and mathematical problem situations.	
5. model and explain part/whole relationships in everyday situations.		

Mathematics Content Standard 3

Students use algebraic concepts, processes, and language to model and solve a variety of real-world and mathematical problems.

Rationale

Algebra is the language of mathematics and science. Through the use of variables and operations, algebra allows students to form abstract models from contextual information.

Benchmarks

Students will:

End of Grade 4	End of Grade 8	Upon Graduation—End of Grade 12
1. use symbols (e.g., boxes or letters) to represent numbers in simple situations.	1. understand the concepts of variable, expression and equation.	1. use algebra to represent patterns of change.
2. explore the use of variables and open sentences to express relationships (e.g., missing addend).	2. represent situations and number patterns using tables, graphs, verbal rules, equations, and models.	2. use basic operations with algebraic expressions.
3. use inverse operations and other strategies to solve number sentences.	3. recognize and use the general properties of operations (e.g., the distributive property).	3. solve algebraic equations and inequalities: linear, quadratic, exponential, logarithmic, and power.
	4. solve linear equations using concrete, numerical and algebraic methods.	4. solve systems of algebraic equations and inequalities, including use of matrices.
	5. investigate inequalities and nonlinear relationships informally.	5. use algebraic models to solve mathematical and real-world problems.

Mathematics Content Standard 4

Students demonstrate understanding of shape and an ability to use geometry.

Rationale

The study of geometry helps students represent and make sense of the world by discovering relationships and developing spatial sense.

Benchmarks

Students will:

End of Grade 4	End of Grade 8	Upon Graduation—End of Grade 12
1. describe, model, and classify two- and three-dimensional shapes.	1. identify, describe, construct, and compare plane and solid geometric figures.	1. construct, interpret, and draw three-dimensional objects.
2. investigate and predict results of combining, subdividing, and changing shapes.	2. understand and apply geometric properties and relationships (e.g., the Pythagorean Theorem).	2. classify figures in terms of congruence and similarity and apply these relationships.
3. identify lines of symmetry, congruent and similar shapes, and positional relationships.	3. represent geometric figures on a coordinate grid.	3. translate between synthetic and coordinate representations.
	4. explore properties and transformations of geometric figures.	4. deduce properties of figures using transformations, coordinates, and vectors in problem solving.
	5. use geometry as a means of describing the physical world.	5. apply trigonometric ratios (sine, cosine and tangent) to problem situations involving triangles.

Mathematics Content Standard 5

Students demonstrate understanding of measurable attributes and an ability to use measurement processes.

Rationale

The first step in scientific investigation is understanding the measurable attributes of objects.

Benchmarks

Students will:

End of Grade 4	End of Grade 8	Upon Graduation—End of Grade 12
1. estimate, measure, and investigate length, capacity, weight, mass, area, volume, time, and temperature.	1. estimate, make, and use measurements to describe, compare, and/or contrast objects in real-world situations.	1. apply concepts of indirect measurements (e.g., using similar triangles to calculate a distance).
2. develop the process of measuring and concepts related to units of measurement, including standard units (English and metric) and nonstandard units.	2. select and use appropriate units and tools to measure to a level of accuracy required in a particular setting.	2. use dimensional analysis to check reasonableness of procedures.
3. apply measurement skills to everyday situations.	3. apply the concepts of perimeter, area, volume and capacity, weight and mass, angle measure, time, and temperature.	3. investigate systems of derived measures (e.g., km/sec, g/cm^3).
4. select and use appropriate tools and techniques.	4. demonstrate understanding of the structure and use of systems of measurement, including English and metric.	4. apply the appropriate concepts of estimates in measurement, error in measurement, tolerance, and precision.
	5. use the concepts of rates and other derived and indirect measurements.	
	6. demonstrate relationships between formulas and procedures for determining area and volume.	

Mathematics Content Standard 6

The students demonstrate understanding of and an ability to use data analysis, probability, and statistics.

Rationale

With society's expanding use of data for prediction and decision making, it is important that students develop an understanding of the concepts and processes used in analyzing data.

Benchmarks

Students will:

End of Grade 4	End of Grade 8	Upon Graduation—End of Grade 12
1. collect, organize, and display data.	1. systematically collect, organize, and describe data.	1. use curve fitting to make predictions from data.
2. construct, read, and interpret displays of data, including graphs.	2. construct, read, and interpret tables, charts, and graphs.	2. apply measures of central tendency and demonstrate understanding of the concepts of variability and correlation.
3. formulate and solve problems that involve collecting and analyzing data.	3. draw inferences, construct, and evaluate arguments based on data analysis and measures of central tendency.	3. select an appropriate sampling method for a given statistical analysis.
4. demonstrate basic concepts of chance (e.g., equally likely events, simple probabilities).	4. construct sample spaces and determine the theoretical and experimental probabilities of events.	4. use experimental probability, theoretical probability, and simulation methods to represent and solve problems, including expected values.
	5. make predictions based on experimental results or probabilities.	5. design a statistical experiment to study a problem and communicate the outcomes.
		6. describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.

Mathematics Content Standard 7

Students demonstrate understanding of and an ability to use patterns, relations and functions.

Rationale

One of the central themes of mathematics is the study of patterns, relations, and functions. Exploring patterns helps students develop mathematical power and instills in them an appreciation for the beauty of mathematics.

Benchmarks

Students will:

End of Grade 4	End of Grade 8	Upon Graduation—End of Grade 12
1. recognize, describe, extend, and create a variety of patterns.	1. describe, extend, analyze, and create a variety of patterns and functions.	1. describe functions and their inverses using graphical, numerical, physical, algebraic, and verbal mathematical models or representations.
2. represent and describe mathematical and real-world relationships.	2. describe and represent relationships with tables, graphs, and rules.	2. analyze the graphs of the families of polynomial, rational, power, exponential, logarithmic, and periodic functions.
	3. analyze functional relationships to explain how a change in one quantity results in a change in another.	3. analyze the effects of parameter changes on the graphs of functions and relations, including translations.
	4. use patterns and functions to represent and solve problems.	4. model real-world phenomena with a variety of functions.
	5. describe functions using graphical, numerical, physical, algebraic, and verbal models or representations.	5. use graphing for parametric equations, three-dimensional equations, and recursive relations.

Mathematics Performance Standards: A Profile of Four Levels

The Mathematics Performance Standards describe students' knowledge, skills, and abilities in the mathematics content area on a continuum from kindergarten through grade twelve. These descriptions provide a picture or profile of student achievement at the four performance levels: advanced, proficient, nearing proficiency, and novice.

Advanced:	This level denotes superior performance.
Proficient:	This level denotes solid academic performance for each benchmark. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
Nearing Proficiency:	This level denotes that the student has partial mastery or prerequisite knowledge and skills fundamental for proficient work at each benchmark.
Novice:	This level denotes that the student is beginning to attain the prerequisite knowledge and skills that are fundamental for work at each benchmark.

Grade 4 Mathematics

Advanced A fourth-grade student at the advanced level in mathematics demonstrates superior performance. He/she:

(a) demonstrates self-motivation and emerging independence as a learner;
(b) accurately selects and uses problem-solving strategies;
(c) presents well-organized solutions and communicates in ways that exceed requirements;
(d) uses whole numbers accurately and fluently to estimate, compute, and determine whether results are accurate and reasonable;
(e) effectively applies basic algebraic concepts and clearly communicates representations in a variety of ways;
(f) examines relationships of shapes in the physical world and makes generalizations;
(g) selects and accurately uses appropriate tools for measurement;
(h) accurately predicts and makes reasonable decisions based on data; and
(i) articulately and fluently communicates representations, analyzes patterns, and clearly describes relationships, and applies them to varied situations.

Proficient A fourth-grade student at the proficient level in mathematics demonstrates solid academic performance. He/she:

(a) selects and effectively uses appropriate problem-solving strategies;
(b) consistently presents organized solutions;
(c) uses whole numbers to estimate, compute, and determine whether results are accurate;
(d) applies basic algebra concepts and consistently communicates representations in a variety of ways;
(e) consistently examines and accurately uses relationships of shapes in the physical world;
(f) determines measurable attributes of objects and selects appropriate tools for measurement;
(g) consistently predicts and makes reasonable decisions based on data; and
(h) consistently uses a variety of patterns and describes their relationships.

Nearing Proficiency A fourth-grade student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficient-level mathematics. He/she:

(a) sometimes selects and uses appropriate problem-solving strategies;
(b) sometimes presents organized solutions, but often with limited supporting information;

(c) uses whole numbers to estimate and compute, and results are usually reasonable;
(d) sometimes applies basic algebraic concepts, but seldom communicates representations;
(e) examines some shapes in the physical world, and sometimes sees relationships;
(f) determines measurable attributes of objects, but does not always select appropriate tools for measurement;
(g) often makes inconsistent predictions and inaccurate decisions based on data; and
(h) uses a limited range of patterns, and sometimes describes relationships within those patterns.

Novice A fourth-grade student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental at each benchmark in mathematics. He/she:

(a) selects and uses only a few problem-solving strategies;
(b) often presents poorly organized solutions, often without supporting information or explanation;
(c) lacks clarity and coherence when communicating mathematical concepts;
(d) uses whole numbers to estimate and compute, but is frequently inaccurate;
(e) sometimes determines whether results are reasonable;
(f) demonstrates a basic algebraic understanding of concrete and symbolic representations, but often misconceptions are present;
(g) describes, models, and classifies some shapes;
(h) determines some measurable attributes of objects, but often does not select appropriate tools for measurement;
(i) sometimes predicts, but often makes inaccurate decisions based on data; and
(j) recognizes and represents a limited range of patterns and describes relationships within those patterns, but is frequently inaccurate.

Grade 8 Mathematics

Advanced An eighth-grade student at the advanced level in mathematics demonstrates superior performance. He/she:

(a) demonstrates self-motivation and independence as a learner;
(b) is accurate and fluent when applying mathematical processes;
(c) effectively uses multiple strategies and extends concepts to new situations;
(d) explores hypothetical questions and articulates valid arguments;
(e) applies and extends rational numbers, proportionality, and algebraic concepts to solve real and theoretical problems;
(f) applies complex measurement and geometric relationships to hypothetical situations;
(g) consistently makes accurate predictions and decisions based on basic probability and statistics; and
(h) recognizes interconnections within and outside mathematics.

Proficient An eighth-grade student at the proficient level in mathematics demonstrates solid academic performance. He/she:

(a) effectively applies mathematical processes correctly to solve a variety of problems;
(b) applies mathematics in a variety of contexts;
(c) uses rational numbers, proportionality, and algebraic concepts to represent and accurately solve mathematical problems;
(d) consistently and accurately uses complex measurement, geometric relationships, and properties to describe the physical world;
(e) formulates logical arguments using appropriate mathematical ideas; and
(f) consistently makes reasonable predictions and decisions based on basic probability and statistics.

Nearing Proficiency An eighth-grade student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficient-level mathematics. He/she:

(a) often uses incomplete and incorrect mathematical processes to solve problems, often inaccurately;
(b) communicates mathematical ideas, but often inaccurately;
(c) makes connections, but does not generalize and often his/her arguments lack appropriate supporting mathematical ideas;
(d) sometimes understands and correctly uses numbers, operations, patterns, relations, and functions;
(e) sometimes uses inaccurate or incomplete representations of rational numbers, proportionality, and algebraic concepts to solve mathematical problems;
(f) sometimes has difficulty recognizing complex measurement and geometric relationships and properties which result in inaccurate solutions; and
(g) makes simple predictions and decisions based on basic probability and statistics.

Novice An eighth-grade student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental to each benchmark in mathematics. He/she:

(a) demonstrates limited and incomplete use of mathematical processes;
(b) communicates mathematical ideas, but they are often limited and incomplete;
(c) sometimes uses numbers, operations, patterns, relations, and functions accurately;
(d) makes only immediate, concrete, mathematical connections;
(e) seldom uses algebraic concepts to solve problems; and
(f) makes simple and inconsistent predictions and decisions, often inaccurately, based on data, and seldom recognizes complex measurement, geometric relationships, or properties.

Upon Graduation Mathematics

Advanced A graduating student at the advanced level in mathematics demonstrates superior performance. He/she:

(a) is self-motivated, an independent learner, and extends and connects ideas;
(b) is accurate, articulate, and effective when applying mathematical processes;
(c) effectively uses multiple strategies, extends concepts to new situations, and skillfully communicates the results;
(d) explores hypothetical questions, uses complex reasoning to articulate valid arguments, and constructs proofs;
(e) uses appropriate technology to apply functions, graphs, and algebraic concepts to solve real and theoretical problems;
(f) applies complex measurement and geometric and algebraic relationships to model a variety of problems and situations;
(g) consistently makes accurate and reasonable predictions and decisions based on data, probability, and statistics; and
(h) recognizes interconnections within and outside mathematics.

Proficient A graduating student at the proficient level in mathematics demonstrates solid academic performance. He/she:

(a) consistently applies mathematical processes correctly to solve a variety of problems and communicate the results;
(b) applies mathematics in a variety of contexts;
(c) consistently uses appropriate technology to apply functions, graphs, and algebraic concepts to solve real and theoretical problems;
(d) uses complex reasoning to formulate logical arguments and proofs using appropriate mathematical ideas;
(e) consistently applies complex measurement and geometric and algebraic relationships to model a variety of problems and situations;
(f) makes reasonable predictions and decisions based on data, probability, and statistics; and
(g) recognizes interconnections within and outside mathematics.

Nearing Proficiency A graduating student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficient-level mathematics. He/she:

(a) applies incomplete and incorrect mathematical processes to solve problems, often inaccurately;
(b) communicates mathematical ideas and sometimes extends them, but often inaccurately;
(c) sometimes understands and uses appropriate technology to apply functions, graphs, and algebraic concepts to solve real and theoretical problems;
(d) sometimes demonstrates difficulty recognizing complex measurement and geometric and algebraic relationships which result in inaccuracies;
(e) sometimes makes predictions and decisions based on data, probability, and statistics, often inaccurately; and
(f) makes connections, but does not generalize or prove them and often his/her arguments lack appropriate supporting mathematical ideas and careful reasoning.

Novice A graduating student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental at each benchmark in mathematics. He/she:

(a) demonstrates limited and incomplete use of mathematical processes and problem-solving strategies;
(b) often uses limited and incomplete reasoning to formulate logical arguments and communicate mathematical ideas;
(c) makes only concrete, mathematical connections;
(d) seldom uses appropriate technology to apply functions, graphs, and algebraic concepts to solve problems;
(e) recognizes, on a limited basis, complex measurement, geometric relationships, and properties; and
(f) makes some predictions and decisions, on a limited basis, based on data, but seldom recognizes statistical or probability concepts.